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THE USE OF GEOINFORMATION TECHNOLOGIES TO VISUALIZE THE RESULTS OF GEODETIC MONITORING OF THE EARTH'S SURFACE DISPLACEMENT

Abstract. The article provides a comparative analysis of seven spatial interpolation methods used in the construction of digital surface models (DEM) in the tasks of geodetic monitoring of man-made deformations. The study is based on observations of the subsidence of the Earth's surface in the mined area of the Zhezkazgan deposit. The data was processed in the Golden Software Surfer software environment, which implements a wide range of interpolation algorithms. The effectiveness of the methods was assessed according to the criteria of accuracy, stability and visual smoothness. The results showed that the Kriging method provides the best quality of modeling by taking into account spatial correlation. A method of complex analysis is proposed, which makes it possible to reasonably choose the optimal interpolation algorithm for engineering and geodetic applications and monitoring systems.

Key words: *geodetic monitoring, spatial interpolation, visualization, digital surface model, the Kriging method, geostatistics, Golden Software Surfer.*

Жер бетінің жылжуын геодезиялық мониторинг нәтижелерін визуализациялау үшін геоақпараттық технологияларды пайдалану

Аңдатпа. Мақалада техногендік деформациялардың геодезиялық мониторингі міндеттерінде беттің цифрлық модельдерін (dem) құру кезінде қолданылатын кеңістіктік интерполяцияның жеті әдісіне салыстырмалы талдау жасалды. Зерттеу Жезқазған кен орнының өңделетін учаскесінде жер бетінің шөгуін бақылау деректеріне негізделген. Деректерді өңдеу интерполяция алгоритмдерінің кең ауқымын жүзеге асыратын Golden Software Surfer бағдарламалық ортасында жүзеге асырылды. Әдістердің тиімділігін бағалау дәлдік, тұрақтылық және визуалды Тегістік критерийлері бойынша жүргізіледі. Нәтижелер Кригинг әдісі кеңістіктік корреляцияны ескере отырып, ең жақсы модельдеу сапасын қамтамасыз ететінін көрсетті. Инженерлік-геодезиялық қосымшалар мен мониторингтік жүйелер үшін оңтайлы интерполяция алгоритмін негізді таңдауға мүмкіндік беретін кешенді талдау әдісі ұсынылған.

Түйінді сөздер: *геодезиялық бақылау, кеңістіктік интерполяция, визуализациялау сандық беттік модель, Kriging тәсілі, геостатистика, Golden Software Surfer.*

Использование геоинформационных технологий для визуализации результатов геодезического мониторинга за сдвижением земной поверхности

Аннотация. В статье выполнен сравнительный анализ семи методов пространственной интерполяции, применяемых при построении цифровых моделей поверхности (ЦМР) в задачах геодезического мониторинга техногенных деформаций. Исследование основано на данных наблюдений за оседанием земной поверхности на подрабатываемом участке Жезказганского месторождения. Обработка данных осуществлялась в программной среде Golden Software Surfer, реализующей широкий спектр алгоритмов интерполяции. Оценка эффективности методов проведена по критериям точности, устойчивости и визуальной гладкости. Результаты показали, что метод Кригинга обеспечивает наилучшее качество моделирования за счет учета пространственной корреляции. Предложена методика комплексного анализа, позволяющая обоснованно выбирать оптимальный алгоритм интерполяции для инженерно-геодезических приложений и мониторинговых систем.

Ключевые слова: *геодезический мониторинг, пространственная интерполяция, визуализация, цифровая модель поверхности, метод Kriging, геостатистика, Golden Software Surfer.*

Introduction

Currently, the geographic information system Surfer Golden Software is used to build graphical images of the functions of two variables. Basically, using this program, surface maps on isolines or contour maps are created. The main advantage of the Surfer program is, first of all, the interpolation algorithms built into it, which make it possible to create high-quality digital surface models based on data unevenly distributed in space. The Kriging method is most often used to represent data related to the image of the Earth's surface.

A digital surface model is traditionally represented as values at the nodes of a rectangular regular grid, the discreteness of which is determined depending on the specific task being solved. To store such values, Surfer uses its own GRD files, which have long become the standard for mathematical modeling packages.

When building a surface, Surfer proceeds from the following principles:

- obtaining an image by superimposing several transparent and opaque graphic layers;

- using special drawing tools, as well as applying text information and formulas to create new and edit old images [1].

Modern tasks of geodetic monitoring of man-made deformations of the earth's surface require not only high measurement accuracy, but also the use of effective methods of mathematical modeling and visualization of spatial data. In the conditions of mining territories of mining enterprises, such as the Zhezkazgan deposit, an important direction is the creation of digital relief models (DEM) that allow detailed analysis of subsidence and deformation of the earth's surface, which in turn will allow forecasting possible collapses.

The choice of spatial interpolation method has a decisive influence on the reliability of geodetic conclusions. An incorrect choice of algorithm can lead to local distortions of the model, underestimation of the amplitude of deformations and erroneous engineering interpretation. Despite the widespread use of software packages that automate the construction of DEM, the question of the optimal choice of the interpolation method remains controversial.

The purpose of the study is based on a comparative analysis of the effectiveness of various spatial interpolation methods implemented in the Surfer software environment in relation to geodetic subsidence monitoring data, and to determine the most reliable algorithm for constructing digital models for engineering and geodetic tasks.

The scientific novelty of using Golden Software Surfer in research lies in the transition from simple visualization of surfaces to the creation of complex digital counterparts of geological and ecological environments using dynamic analysis tools. The scientific novelty of using Golden Software Surfer in research lies in the transition from simple visualization of surfaces to the creation of complex digital counterparts of geological and ecological environments using dynamic analysis tools.

The main aspects of novelty include:

- integration of 3D drilling data with surface models: the latest versions allow not only to visualize boreholes, but also to create contact surfaces directly in 3D based on lithological data or complex queries. This provides a qualitatively new level of interpretation of stratigraphy and mineralization in a single space;

- multi-touch lighting and detailing: Using up to four independent light sources in 3D models allows you to identify microrelief features and structural anomalies that were previously hidden in shadows during standard rendering;

- direct work with georeferentiated data: the novelty lies in the possibility of seamlessly superimposing analytical grids on high-resolution satellite images (for example, Google Maps) directly in the Golden Software Surfer environment, which eliminates binding errors when combining data;

- two-way data exchange with BIM and CAD: support for exporting 3D geometry to DXF format while preserving Z parameters allows you to use the results of Surfer geostatistical analysis as a design basis for engineering modeling in AutoCAD [1].

Materials and research methods

The research methods in Golden Software Surfer are based on geostatistics and mathematical modeling of spatial data.

Main groups of methods:

1. Methods of spatial interpolation (Gridding):

- Kriging: the «gold standard» of geostatistics. It allows not only to build a map, but also to estimate a statistical error (variagram), which is critically important for scientific reliability;

- Radial Basis Function: one of the most accurate methods for modeling smooth surfaces and complex hydrogeological fields;

- Inverse Distance to a Power a classic deterministic method for fast visualization of local anomalies.

***2. Geometric and volumetric analysis**:*

- Grid Calculus method: calculation of derivatives (slopes, aspects) and second derivatives (curvature), which is necessary for the analysis of runoff and slope stability;

- Grid Volume: numerical integration methods (trapezoid rules, Simpson's rules) for precision calculation of volumes between two surfaces (for example, production volumes or mineral reserves).

3. Geostatistical analysis (Variogram Graphics):

- The study of spatial autocorrelation of data, the method allows us to scientifically substantiate at what distance data points cease to influence each other.

4. Digital Terrain Modeling (DEM) method:

- Creating multi-layered models where vector layers (flow directions) are superimposed on raster grids, allowing for cross-factor analysis.

The main stages of creating a geostatistical model include:

- data analysis and preprocessing (identification of trends and areas of spatial heterogeneity, analysis of distribution, anisotropy);

- calculation of empirical variogram or covariance values;

- construction of a variogram or covariance model;

- obtaining the prognostic value and the error (uncertainty) of the estimate at any point of the studied territory [2].

When creating a contour map, in addition to the usual controls for displaying isolines, axes, frames, markers, legends, and other features, it was possible to create maps by filling individual zones with color or different types of lines. A contour map is usually referred to as an «isolated map». Visualization of geodetic observations of the Earth's surface movement in the form of contour maps, vector maps, DEM, superimposing combinations of various maps, wireframe maps of the studied area of the Earth's surface based on instrumental observations.

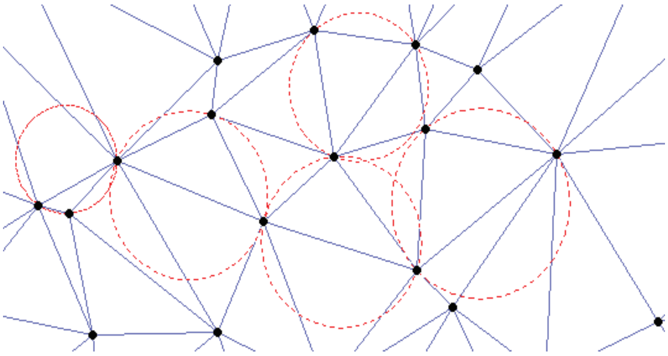
The studies for the construction of the DEM were carried out using six spatial interpolation methods: Inverse Distance to a Power method, Modified Shepard's Method, Triangulation with Linear Interpolation, Natural Neighbor method, Minimum Curvature method, Radial Basis Function Method, Kriging Methods [3].

Each of the interpolation methods has its own set of parameters. To describe the behavior of spatially distributed phenomena, the concept of a spatial variable is used – it is a numerical variable $Z = Z(X, Y)$, which takes on a value at any point in the studied area; X, Y are the coordinates of a point in space (on a plane). For example, the content of heavy metals in the upper soil layer, the height of the relief, and the thickness of the geological layer can be considered as spatial variables in two-dimensional space [4].

The Inverse Distance to a Power method. As an estimate of the variable Z at some point in the study area, this method uses the weighted average of the values of the variable Z at the sample points. The inverse distance method works on the assumption that experimental values measured close to the interpolated position have a greater impact on the evaluation result than those that are far from it. Therefore, it is necessary to give more weight to the sampling points closest to the point where the estimate is made; the weight of the sampling points should decrease as the distance from the interpolated position increases.

Modified Shepard's Method, which works under the same assumptions as the inverse distance method, but reduces «the bull's-eye effect». The weighted average of the approximation functions at a given point is used as an estimate of the variable Z at any point (X_0, Y_0) of the studied area.

Triangulation with Linear Interpolation. This method uses the Delaunay triangulation algorithm to construct a division of the studied area into a set of triangles. This is a method of constructing a digital surface model (DMP) by connecting the source points into a network of triangles (TIN), inside each of which the value is interpolated along an inclined plane defined by the vertices. This precise interpolator ensures that the values at the nodes match, and is suitable for data with sharp discontinuities, but it can create a «faceted» surface (Figure 1) [5].



Сурет 1. Delone триангуляциясы.
Рис. 1. Триангуляция Делоне.
Figure 1. Delaunay triangulation.

The vertices of the triangles are adjacent sampling points; the points are connected so that the sides of the triangles do not intersect. None of the triangulation points fall within the circle described around any constructed triangle. Using this method with a small number of sampling points leads to the appearance of obvious triangular faces on the raster surface of the model or large rectilinear segments on the contour map [6].

The estimate of the variable $\boldsymbol{Z}$ at the point ($\boldsymbol{X_0}$, $\boldsymbol{Y_0}$) will be the corresponding value on this plane:

$$\widehat{Z_0} = \frac{-a \cdot X_0 - b \cdot Y_0 - d}{c},$$

where $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ – coefficients.

Linear interpolation triangulation works best if the data is evenly distributed over the study area.

Building a digital model of the Earth's surface (DEM/DTM) based on Delaunay triangulation allows you to create a TIN model (Triangulated Irregular Network), an irregular network of triangles that describes complex terrain as accurately as possible with minimal data.

The Minimum Curvature method. Under this name, Surfer uses a 2D spline with tension (minimizing curvature). The surface constructed by this method is like a thin elastic plate, tending to pass through all experimental points with a minimum number of bends. The method is not an accurate interpolator, it generates the smoothest possible surface that comes closest to the experimental points. The resulting $\boldsymbol{Z}$ estimate is obtained by summing the polynomial approximating surface and the interpolated residuals [4–8].

Unlike all the methods described above, which belong to the deterministic class, the Kriging method is a geostatistical method. Geostatistical methods are based on a probabilistic model that considers the spatial variable $\boldsymbol{Z}$ ($\boldsymbol{X}$, $\boldsymbol{Y}$) under study as a realization of a random function $\boldsymbol{Z}$ ($\boldsymbol{X}$, $\boldsymbol{Y}$) [6].

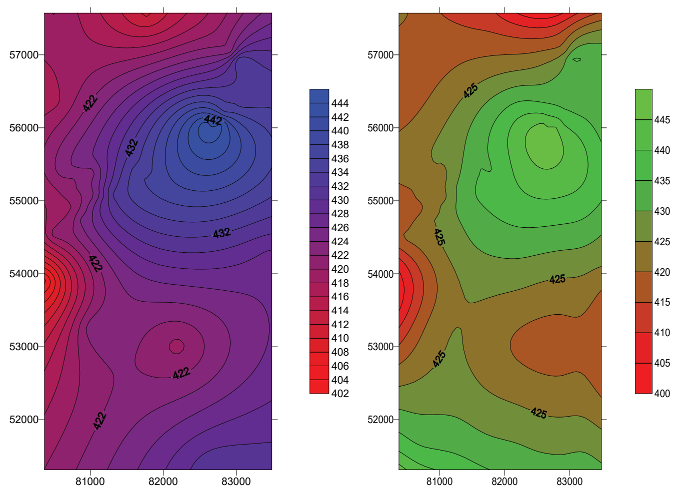
This approach allows you to take into account the spatial correlation of data and makes it possible not only to create surface models, but also to evaluate the accuracy of these models. The geostatistical interpolation method is based on statistical analysis. Statistical techniques allow you to create surfaces of preliminary values and interpret the degree of reliability of these preliminary calculations.

Results

The data obtained by the high-precision leveling method for 2015, 2018 and 2020 years as the initial data for the Zhezkazgan deposit under development. The results of geodetic monitoring showed the spatial variability of the subsidence of the Earth's surface in the mined area of the Zhezkazgan deposit.

In 2020, the area of maximum subsidence shifted to the southeastern part of the site, which indicates a gradual intensification of processes in the field's re-mining area. The greatest elevation changes are observed in the central part of the profile, where the main production workings are located, while the peripheral areas are characterized by relative terrain stability.

Figure 2 shows contour maps constructed using the Kriging method and the minimum curvature method based on 2020 year, demonstrating the distribution of deformation zones.



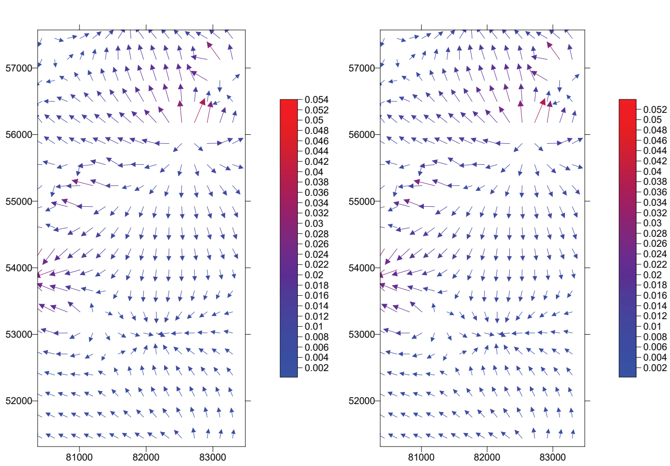
a *b*

a – the Kriging method; b – the minimum curvature method
Figure 2. Contour maps based on geodetic subsidence monitoring data from the Zhezkazgan deposit.
а – Kriging әдісі; б – минималды қисықтық әдісі
Сурет 2. Жезқазған кен орнының шөгуінің геодезиялық мониторингі деректері бойынша контурлық карталар.
а – метод Kriging; б – метод минимальной кривизны
Рис. 2. Контурные карты по данным геодезического мониторинга оседаний Жезказганского месторождения.

On the contour map, the zones of subsidence and rise of the displacement values of the ground reference points are expressed in color. The vector map shows the direction and rate of decrease of the z-indicators of the reference points, and the arrows on the vector map indicate the downward direction, and the length of the arrows corresponds to the size (steepness) of the reference point. tilt.

A comparison of the methods made it possible to identify not only the absolute values of subsidence, but also their directional displacement, which is important for predicting further deformation processes.

Figure 3 shows contour and vector models of the surface constructed by various interpolation methods.



a *b*

a – the IDW method; b – the Shepard method

Figure 3. Contour and vector models of the surface constructed by various interpolation methods.

а – IDW мәсілі; б – Shepard мәсілі

Сурет 3. Интерполяцияның әртүрлі әдістерімен салынған беттің контурлық және векторлық модельдері.

а – метод IDW; б – метод Shepard

Рис. 3. Контурные и векторные модели поверхности, построенные различными методами интерполяции.

A comparison of the results of various spatial interpolation algorithms was carried out on the basis of contour and vector models reflecting the distribution of subsidence of the Earth's surface. The methods of inverse distance weighting (IDW), Shepard and triangulation implemented in the Golden Software Surfer software environment were used for the analysis [7].

Overlaying a vector layer onto a contour map or 3D surface allows you to see how the flows relate to the terrain. To increase the visibility and comprehensive interpretation of the results of spatial interpolation, the contour and frame models of the site surface were combined (Figure 4). This overlay allows you to simultaneously visualize the distribution of subsidence and the geometric structure of the relief, providing a three-dimensional perception of spatial changes.

The use of a combined approach increases the information content of the analysis, as it allows us to identify the relationship between areas of intense subsidence and relief elements. At the same time, the Kriging method demonstrated the optimal balance between smoothness and accuracy of surface reconstruction, correctly interpolating areas of local deformations and ensuring a minimum value of the RMS error (RMSE ≈ 0,006 м) [8].

The use of overlay models is recommended in engineering and geodetic research and monitoring systems, where integration of various types of spatial data and visual comparison of modeling results with the geometry of the terrain are required.

The standard error map shows minimal values in areas with a dense observation network, where the main reference points are located, and a gradual increase in errors to the periphery of the site. This confirms the correctness of the applied interpolation model and the high predictive reliability of the Kriging method in assessing relief deformations. The results obtained indicate that the geostatistical structure of subsidence corresponds to the physical nature of the process and ensures sufficient stability of the model for practical application in engineering and geodetic monitoring.

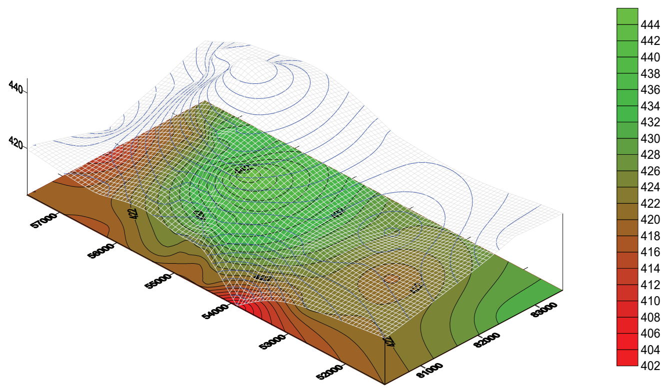


Figure 4. Overlay of contour and wireframe models of the Earth's surface, built using Surfer GIS.

Сурет 4. Жер бетінің контурлық және рамалық модельдерінің қабаттасуы, surfer ГАЖ көмегімен салынған.

Рис. 4. Оверлей контурной и каркасной моделей земной поверхности, построенные с помощью ГИС Surfer.

Discussion of the results

The results of the study showed that the use of geostatistical interpolation methods allows not only to reconstruct the subsidence surface with high accuracy, but also to quantify the uncertainty of the model. The Kriging method, unlike deterministic approaches, takes into account spatial correlation, anisotropy and trends of the initial data, which makes it the most preferred tool for engineering and geodetic monitoring of deformations.

The use of combined approaches, such as pre-smoothing by the radial basis functions (RBF) method followed by refinement by kriging approximation, further improves the accuracy of modeling. Error analysis showed that with an irregular distribution of benchmarks, the discrepancy between the RBF and Kriging results does not exceed 10%, however, it is the kriging model that remains an unbiased predictor and provides confidence intervals for estimating spatial error [9].

Conclusion

The Kriging method provides an optimal balance between accuracy and smoothness of the surface when modeling subsidence and allows you to estimate the spatial uncertainty of the forecast. According to the analysis results, the standard deviation was RMSE ≈ 0.006m, which confirms the high reliability of the model.

For applied engineering and geodetic tasks, it is recommended to use the Kriging method in conjunction with standard error maps and cross-validation procedures, which provides an objective assessment of the reliability of the results. The prospects for further research are related to the consideration of the anisotropy of subsidence, variations in the geological structure and the development of adaptive models combining geostatistical and neural network approaches to the prediction of deformations.

The Kriging method is a very flexible method, it can be either an accurate or a smoothing interpolator, depending on the parameter values. It allows you to efficiently and naturally incorporate anisotropy and trends into the model. If we consider only interpolation methods that work on the basis of weighted averaging, then the Kriging method is the best unbiased predictor. If the data has a normal distribution, then the Kriging method becomes the best predictor among all unbiased predictors [10].

When visualizing quantities using the Kriging method, good results are obtained for most data, and therefore it is recommended to use this method to visualize geodetic monitoring data for the displacement of the Earth's surface during the development of mineral deposits.

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