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APPLICATION OF THE KRIGING METHOD FOR GRAVITY DATA INTERPOLATION

Abstract. This article examines the application of the kriging method for the interpolation of gravimetric data. Three approaches are analyzed – Ordinary Kriging, Universal Kriging, and Empirical Bayesian Kriging – with the aim of constructing spatially continuous models of gravity anomalies. The methodology is implemented using the ArcGIS Pro toolkit. Particular attention is given to comparing the interpolation accuracy based on cross-validation indicators, including root mean square error and mean prediction error. The results show that EBK and UK with a linear trend provide the highest accuracy under various data characteristics, especially in the presence of trends and sparse observations. Kriging demonstrates high robustness, interpretability, and adaptability to the spatial distribution of gravimetric measurements, making it an effective tool in gravimetry and geoid modeling.
Key words: *gravimetry, kriging, spatial interpolation, variogram, geostatistics, geoid, gravitational field.*

Гравиметриялық деректерді интерполяциялау үшін крикинг әдісін қолдану

Аңдатпа. Бұл мақалада гравиметриялық деректерді интерполяциялау үшін крикинг әдісін қолдану қарастырылады. Гравитациялық аномалиялардың кеңістіктік үздіксіз модельдерін құру мақсатында үш тәсіл талданады – Ordinary Kriging, Universal Kriging және Empirical Bayesian Kriging. Әдістеме ArcGIS Pro бағдарламалық құралы негізінде жүзеге асырылған. Интерполяция дәлдігін кросс-валидация көрсеткіштері (орташа квадраттық қате мен болжам қатесінің орташа мәні) бойынша салыстыруға ерекше назар аударылады. Нәтижелер EBK және сызықтық тренді бар UK әдістері әртүрлі деректер сипаттамаларында, әсіресе трендтер мен сирек өлшемдер жағдайында, жоғары дәлдік қамтамасыз ететінін көрсетеді. Крикинг әдісі гравиметриялық өлшемдердің таралу ерекшеліктеріне жоғары тұрақтылық, түсініктілік және бейімділік танытады, бұл оны гравиметрия мен геоидты модельдеуде тиімді құрал етеді.
Түйінді сөздер: *гравиметрия, крикинг, кеңістіктік интерполяция, вариограмма, геостатистика, геоид, гравитациялық өріс.*

Применение метода кригинг для интерполяции гравиметрических данных

Аннотация. В статье рассматривается применение метода крикинга для интерполяции гравиметрических данных. Исследуются три подхода – Ordinary Kriging, Universal Kriging и Empirical Bayesian Kriging – с целью построения пространственно-непрерывных моделей гравиметрических аномалий. Методология реализована с использованием инструментария ArcGIS Pro. Особое внимание уделено сравнению точности интерполяции на основе показателей кросс-валидации, включая среднеквадратичную ошибку и среднюю ошибку предсказания. Результаты показывают, что EBK и UK с линейным трендом обеспечивают наилучшую точность при различных характеристиках данных, особенно в условиях наличия трендов и разреженных наблюдений. Метод крикинга демонстрирует высокую устойчивость, интерпретируемость и адаптивность к особенностям распределения гравиметрических измерений, что делает его эффективным инструментом в гравиметрии и моделировании геоида.
Ключевые слова: *гравиметрия, крикинг, пространственная интерполяция, вариограмма, геостатистика, геоид, гравитационное поле.*

Introduction

Kriging is one of the most effective geostatistical interpolation methods, widely used in gravimetry for modeling the gravity field and constructing geoid models. Its popularity stems from its ability to account for the spatial autocorrelation of data and to provide optimal estimates with minimal error variance [1]. Modern software platforms such as ArcGIS Pro offer various kriging techniques, including Ordinary Kriging, Universal Kriging, and Empirical Bayesian Kriging. These methods allow for adaptation to different data characteristics, including trends and heterogeneities, making them particularly valuable for analyzing gravity anomalies and constructing geoid models[1].

Recent research on the application of machine learning methods for predicting gravity anomalies highlights the potential of these approaches to enhance interpolation accuracy, particularly in regions with sparse or irregular observation networks. For instance, Zhanakulova et al. (2025) compared the performance of various machine learning algorithms with traditional interpolation techniques, including kriging, and found that in mountainous regions, kriging provided higher accuracy and greater stability than other methods[2]. Luther (2025) examined the performance of machine learning and kriging in modeling gravity anomalies. When a sufficient amount of training data was available, neural network models – such as XGBoost and CNN –demonstrated superior accuracy in capturing complex patterns. However, kriging proved to be more robust to local noise and offered better interpretability. Wang et al. (2019) proposed using convolutional neural networks to reconstruct gravity and magnetic data with large missing areas. While neural networks were successful in handling datasets with significant gaps, kriging yielded better results in cases with fewer missing values and the presence of spatially autocorrelated noise [2].

At the same time, foundational works in the field of geodesy – such as the study by Torge and Müller (2012) – emphasize the importance of accurately modeling the Earth's gravity field and the role of various interpolation methods in this process. The authors examine different aspects of gravimetric measurements and their application in geoid modeling, highlighting the relevance of selecting an appropriate interpolation method depending on the specific characteristics of the task at hand [3]. Tóth and Völgyesi (2006) investigated the capabilities of kriging and least-squares collocation (LSC) for predicting gravity field values based on gradient data. They concluded that under limited resource conditions, kriging is preferable, as it requires less computational effort and is more robust to sparse observation networks [4].

The choice of interpolation method – whether traditional kriging or modern machine learning approaches – should be guided by the characteristics of the input data, the required level of accuracy, and the specific nature of the problem being addressed. An integrated approach that leverages the strengths of various methods can yield optimal results in gravity field modeling and geoid construction [5, 6, 7].

Fedorchuk (2024) conducted a comparative analysis of different interpolation techniques using data from WGM2012. The study found that kriging provides high accuracy when constructing regular grids of gravity anomalies, particularly when data points are evenly distributed. Kriging outperformed

[1]*ESRI. (2022). Kriging methods in ArcGIS Pro [Help documentation]. URL: https://pro.arcgis.com (retrieved May 29, 2025). Access mode: open.*
[2]*K. Zhanakulova. Application of machine learning methods for gravity anomaly prediction // Geosciences. 2025. V. 15. No. 5. Article 175. URL: https://doi.org/10.3390/geosciences15050175 (retrieved May 29, 2025). Access mode: open.*

inverse distance weighting, minimum curvature, and radial basis function methods, especially in scenarios with moderate to high-density GNSS-gravimetric observations. The author emphasizes that kriging serves as a reliable tool for local refinement of geoid models based on global models such as EGM2008 and WGM2012, particularly in mountainous and transitional regions where topographic effects are most pronounced [8].

Yang et al. (2024) applied kriging to assess the resolving power of satellite-derived gravity anomaly models. Their research demonstrated that kriging enables local refinement of global datasets (e.g., DTU17 and SIOv32.1), especially in areas where satellite data lack sufficient density and precision[3]. In this context, kriging was essential for restoring residual anomalies and estimating the effective elastic thickness of the lithosphere, thereby facilitating the interpretation of uplift mechanisms in active tectonic regions.

Liu et al. (2019) developed a modified kriging technique incorporating «topographic weighting» to enhance accuracy in marine gravimetry. This method significantly improved results in areas with limited bathymetric data and substantially reduced discrepancies when reconciling satellite altimetry data with navigational measurements [9].

Materials and Methods

One of the most widely applied approaches to spatial interpolation is geostatistical modeling based on the kriging method. Kriging is an optimal linear unbiased interpolator that relies on the assumption of spatial autocorrelation among observed values. It provides weighted predictions in continuous space, allowing for the incorporation of both the structure of spatial variability and the level of confidence in the input data.

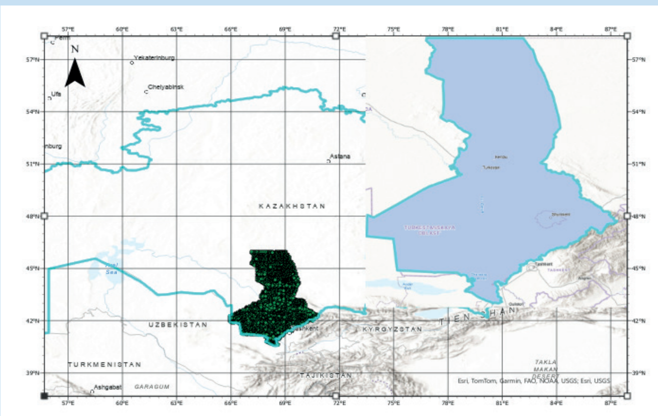

Figure 1. Study Area Map.
Сурет 1. Зерттеу аумағының картасы.
Рис. 1. Карта территории исследования.

In kriging, the predicted value at any location is computed as a linear combination of the values obtained at sampled points, with corresponding weights. These kriging weights are selected to minimize the variance of the predicted value. They depend on the choice of the variogram model, the distances between the measured and predicted points, and the overall spatial structure of the data distribution.

Ordinary kriging is a widely used interpolation method that assumes a stationary random process with a known semivariogram and an unknown but constant mean, which is either estimated or ignored in the modeling process. It ensures unbiased estimation of spatial variables.

The study area is presented on the map and covers the Turkistan Region of the Republic of Kazakhstan, where digitized gravity anomaly measurement points were collected for subsequent analysis.

As part of this study, three types of kriging were applied: Ordinary Kriging, Empirical Bayesian Kriging, and Universal Kriging. All of these methods are based on variational analysis, but they differ in their approaches to variogram modeling and in how they handle uncertainty in model parameters.

Ordinary Kriging assumes first-order stationarity, meaning that the mean is constant but unknown throughout the study area. Constructing an interpolation model requires a priori selection and parameterization of a theoretical variogram model – commonly spherical, exponential, or Gaussian. The variogram describes how the variance between values changes with distance and serves as the foundation for computing interpolation weights. Key parameters include the sill (the variance at which the variogram levels off), the range (the distance beyond which spatial correlation becomes negligible), and the nugget effect (representing microscale variation or measurement error). This method is sensitive to the choice of variogram model and typically requires manual adjustment, particularly in the presence of anisotropy or spatial trends in the data.

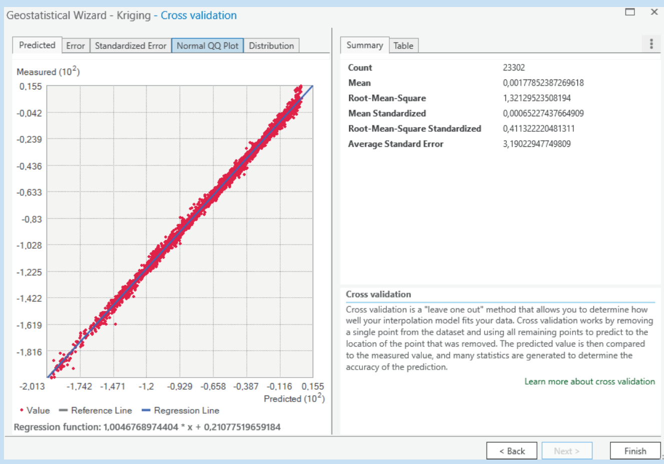

Figure 2. Kriging cross-validation results.
Сурет 2. Крикинг әдісінің кросс-валидация нәтижелері.
Рис. 2. Результаты кросс-валидации метода крикинга.

Empirical Bayesian Kriging (EBK) is a modification of the classical kriging approach in which variogram parameters are treated as random variables and are estimated through repeated simulations. The method employs a Bayesian approximator that allows for the automatic incorporation of uncertainty in modeling spatial correlation. This makes EBK particularly well-suited for handling heterogeneous, noisy, or sparsely

[3] *Yang, D., Zhang, L., & Chen, X. Using kriging to assess resolution of satellite gravity models. Remote Sensing. 2024. V.16(3). Article 423. URL: https://doi.org/10.3390/rs16030423 (retrieved May 29, 2025). Access mode: open.*

sampled datasets – conditions frequently encountered in the interpolation of gravity anomalies.

EBK eliminates the need for manual variogram fitting and can deliver more robust results under similar conditions. Its ability to model parameter uncertainty directly enhances its performance in cases where classical kriging methods may be less reliable due to data limitations or non-stationarity.

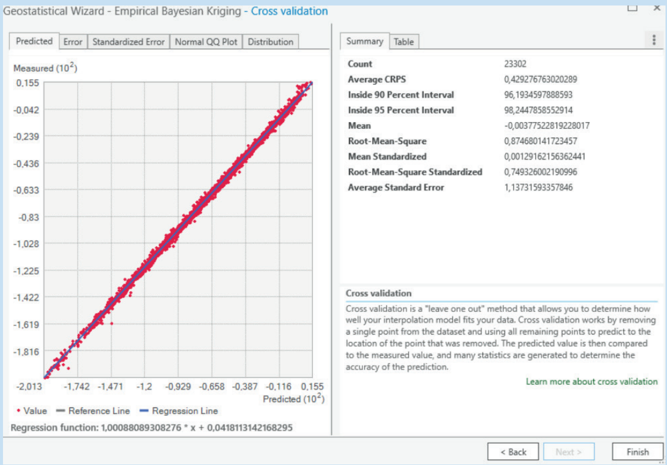

Figure 3. Empirical Bayesian kriging cross-validation results.
Сурет 3. Эмпирикалық байес крикингі әдісінің кросс-валидация нәтижелері.
Рис. 3. Результаты кросс-валидации метода Эмпирический байесовский крикинг.

Universal Kriging (UK) is a variant of kriging that accounts for the presence of a global trend in the spatial distribution of data. Unlike Ordinary Kriging, which assumes a constant mean throughout the study area, Universal Kriging allows the mean to vary – typically as a linear or polynomial function of spatial coordinates.

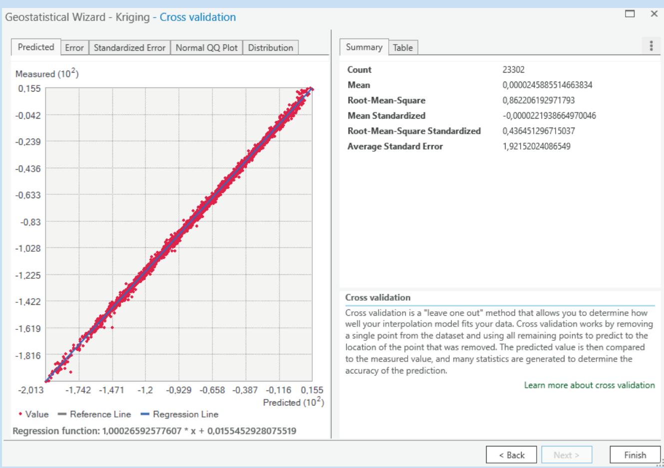

Figure 4. Universal Kriging cross-validation results.
Сурет 4. Универсалды крикингі әдісінің кросс-валидация нәтижелері.
Рис. 4. Результаты кросс-валидации метода универсального крикинга.

Universal Kriging with a linear trend represents an optimal compromise between deterministic and stochastic modeling, offering high-quality interpolation of gravimetric data in the presence of weak global directional trends. This makes UK particularly suitable for scenarios where the data exhibit gradual spatial drift or underlying systematic variation across the region of interest.

The methods under consideration are based on linear interpolation models and, on average, provide accurate approximations of values without systematic bias. However, they differ in terms of automation, trend incorporation, and robustness to sampling characteristics. A comparative analysis of these methods using the same set of gravimetric data enables not only the evaluation of differences in spatial prediction accuracy but also a reasoned justification for selecting the most appropriate geostatistical model for subsequent applications in gravimetry and geoid modeling.

In spatial analysis, three functional models are commonly used: spherical, exponential, and Gaussian. In this study, the spherical model was selected, and is represented by the following equation:

$$\gamma(h) = c_0 + c\left\{\frac{3h}{2r} - \frac{1}{2}\left(\frac{h}{r}\right)^3\right\} \text{ for } 0 < h \le r = c_0 + c$$

$$\text{for } h > r = 0 \text{ for } h = 0,$$

where

c – spatially correlated variance;

r is the correlation radius.

The quantity $c_0 + c$ is known as the «threshold».

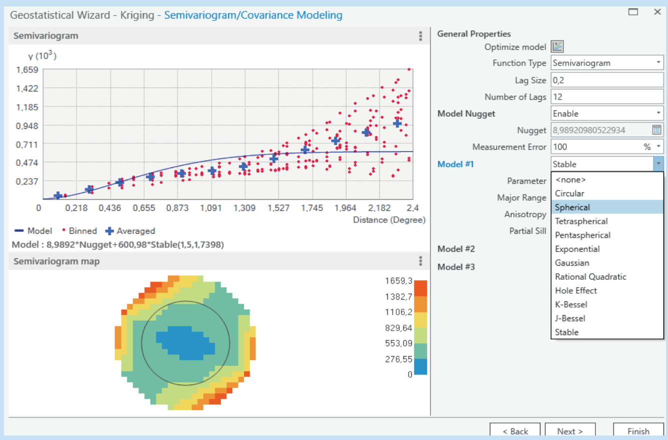

Figure 5. Selection of the Spherical Variogram Model.
Сурет 5. Сфералық вариограмма моделін таңдау.
Рис. 5. Выбор сферической вариограммной модели.

Results and discussion

As part of the study, three geostatistical interpolation methods were tested: Ordinary Kriging (OK), Empirical Bayesian Kriging (EBK), and Universal Kriging (UK) with a first-order trend. All methods were implemented in the ArcGIS Pro software environment using a digitized database of Bouguer anomalies, followed by validation through cross-validation.

Table 1
Comparative Analysis of Kriging Methods Based on Cross-Validation Metrics for Gravity Anomaly Interpolation
Кесте 1
Гравитациялық аномалияларды интерполяциялау үшін кросс-валидация метрикалары негізінде кригаж әдістерінің салыстырмалы талдауы
Таблица 1
Сравнительный анализ методов кригинга на основе метрик перекрестной проверки для интерполяции аномалий гравитации

Interpolation Method	Root Mean Square Error (RMSE)	Standardized RMSE	Average Standard Error (ASE)	Comment
Ordinary Kriging (OK)	1.3213	0.4113	3.1902	Simple and robust model without trend consideration
Empirical Bayesian Kriging (EBK)	0.8749	0.7493	1.1373	Automated model accounting for local variability
Universal Kriging (UK)	0.8622	0.4365	1.9215	Takes into account the global trend, provides a high level of accuracy and stability

The simplest model – Ordinary Kriging (OK) – demonstrated stable but less accurate results (RMSE = 1.32). The standardized RMSE was 0.41, indicating a moderate underestimation of variance and an acceptable prediction error. As this method does not account for a global trend in the spatial data, it is most suitable for cases with isotropic distributions of anomalies.

Empirical Bayesian Kriging (EBK) achieved higher accuracy (RMSE = 0.87) and offered a better balance between predicted values and estimated errors. The automatic generation of semivariograms and the inclusion of localized models make EBK particularly effective under conditions of weak stationarity and heterogeneous spatial structure.

Universal Kriging (UK) with a first-order trend produced the highest interpolation quality among the methods tested (RMSE = 0.86), with a standardized RMSE of 0.43. This suggests that incorporating a global linear trend allows for the modeling of systematic variations in Bouguer anomalies across the study area without compromising physical interpretability. However, applying a second-order trend led to model overfitting and a loss of geostatistical significance, as evidenced by an extremely high standardized error (exceeding 37). Therefore, this variant was excluded from further analysis.

In conclusion, EBK and UK with a first-order trend are the most optimal approaches for the spatial interpolation of gravimetric data in this study. The choice between them should be guided by the analytical objective: Universal Kriging is more appropriate for modeling global spatial patterns, whereas Empirical Bayesian Kriging provides more robust results in the presence of local heterogeneity.

Conclusion

The conducted study confirms that kriging remains one of the most reliable and accurate tools for the spatial interpolation of gravimetric data. The analysis of its three variants – Ordinary Kriging, Universal Kriging, and Empirical Bayesian Kriging – highlighted their respective strengths depending on the structure of the input data. The highest accuracy under conditions of spatial trends and heterogeneity was achieved by EBK and UK. EBK proved particularly effective when the number of observations was limited, while UK excelled in the presence of a global trend.

Kriging provides statistically sound estimates, demonstrates robustness to noise, and adapts well to various spatial characteristics. These features affirm its applicability as a key technique in gravity field modeling and the construction of refined geoid models, especially in regions with sparse observation networks and complex topography.

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