

Код МРНТИ 52.13.25:52.01.77

V.V. Zhurov, *R.A. Mussin, N.M. Zamaliyev, D.R. Akhmaturov

NJS «Karaganda State Technical University named after Abylkas Saginov» (Karaganda, Kazakhstan)

COMPUTER MODELING OF THE STRESS-STRAIN STATE OF THE NEAR-CONTOUR ROCK MASS

Abstract. The article examines the modeling of the stress-strain state (SSS) of the near-contour rock mass during the preparation and operation of mine workings. The relevance of the topic is associated with the need to improve the safety and efficiency of mining operations at Qarmet JSC enterprises. Calculations of the SSS of the rock mass were performed using metal-arch and anchor supports. The influence of anchor length and diameter on stress distribution was established: longitudinal stresses increase with their growth, while shear stresses change insignificantly. Patterns of stress growth depending on the thickness of roof rocks were identified. The research results, validated by industrial testing, demonstrate opportunities to enhance the stability of workings and develop adaptive support technologies for various mining and geological conditions.

Key words: rock pressure, ventilation, air circulation, stress, anchor support, mining safety, stress modeling, mine workings, near-contour rock mass, geomechanics.

Компьютерлік модельдеу: қаптал массивінің кернеулі-деформацияланған күйі

Аннотация. Мақалада тау-кен қазбаларын дайындау және пайдалану кезінде контур маңы тау жыныстары массивінің кернеулі-деформацияланған күйін (КДК) модельдеу қарастырылады. Тақырыптың өзектілігі «Qarmet» АҚ-ның ТҚК кәсіпорындарында тау-кен жұмыстарын тиімділігі мен қауіпсіздігін арттыру қажеттілігімен байланысты. Метал доғалы және анкерлік бекітпелерді пайдалана отырып, массивтің КДК есептеулері жүргізілді. Анкерлердің ұзындығы мен диаметрінің кернеулерге әсері анықталды: ұзындығы мен диаметрі артқан сайын бойлық кернеулер ұлғаяды, ал жанама кернеулер айтарлықтай өзгермейді. Төбе жыныстарының қалыңдығына байланысты кернеулердің өсу заңдылықтары анықталды. Зерттеу нәтижелері өндірістік сынақтармен расталды, қазбалардың тұрақтылығын арттыру және әртүрлі тау-геологиялық жағдайлар үшін бейімделген бекіту технологияларын әзірлеу мүмкіндіктерін көрсетті.

Түйінді сөздер: тау қысымы, желдету, ауа алмасу, кернеу, анкерлік бекітпе, тау-кен жұмыстарының қауіпсіздігі, кернеулерді модельдеу, тау-кен қазбалары, контур маңы массиві, геомеханика.

Компьютерное моделирование напряженно-деформированного состояния приконтурного массива

Аннотация. В статье исследуется моделирование напряженно-деформированного состояния (НДС) приконтурного массива пород при подготовке и эксплуатации шахтных выработок. Актуальность темы связана с необходимостью повышения безопасности и эффективности горных работ на предприятиях УД АО «Qarmet». Проведены расчеты НДС массива с использованием металлоарочной и анкерной крепи. Установлено влияние длины и диаметра анкеров на характер напряжений: продольные растут с их увеличением, а касательные изменяются незначительно. Выявлены закономерности роста напряжений в зависимости от мощности пород кровли. Результаты исследований подтверждены производственными испытаниями, демонстрируют возможности повышения устойчивости выработок и разработки адаптивных технологий крепления для различных горно-геологических условий.

Ключевые слова: горное давление, вентиляция, проветривание, напряжение, анкерная крепь, безопасность горных работ, моделирование напряжений, горные выработки, приконтурный массив пород, геомеханика.

Introduction

The modern coal mining industry faces numerous challenges related to the efficiency and safety of underground workings. Increasing production volumes and ensuring the stability of operations demand innovative approaches to designing and supporting mine workings. Instabilities in the rock masses surrounding mine workings result in significant costs for repairs, operational delays, and safety risks. Advances in computer modeling enable researchers to analyze the stress-strain state (SSS) of rock masses with high accuracy, leading to better predictions of rock behavior and optimized support designs. This study focuses on developing technologies for stable and safe mine workings, considering the specific mining and geological conditions of JSC «Qarmet» mines.

In the work of Protosenya et al. [1], approaches to modeling the SSS of block rock masses near individual mine workings and in the influence zone of rock beams are analyzed. The authors emphasize the importance of accounting for the structural heterogeneity of the rock mass and its mechanical properties when predicting the stability of excavations.

Gospodarikov and Zatsepin [2] present mathematical modeling of the SSS of rock masses in the development of shallow-angle deposits. Their research demonstrates the application of numerical methods to evaluate the impact of mining operations on the state of the rock mass, allowing optimization of support parameters and improving the safety of underground operations.

The textbook «Mine Support for Underground Workings in Mines and Quarries» [3] explores modern schemes and methods for constructing vertical workings and the equipment

used in these processes. The authors provide techniques for the techno-economic justification of excavation schemes, equipment selection, support types, and optimization of excavation cycles. Special attention is paid to rock mechanics, which is essential for solving tasks related to excavation and support.

Thus, the use of modern geomechanical modeling methods and consideration of the specific conditions of rock mass formation are crucial factors in developing efficient and safe technologies for underground mining operations. This study aims to contribute to the existing scientific literature, confirming the importance of an integrated approach to analyzing the SSS and optimizing support parameters in complex geological conditions.

Research Methods

The study employed advanced numerical simulation software to model the stress-strain state (SSS) of the rock masses surrounding underground mine workings. Key parameters, such as roof thickness, rock layer composition, and stress concentration zones, were included in the models. Both metal arch and rock bolt supports were analyzed under varying conditions. The simulations examined the effects of changes in anchor length (1.8–2.4 m) and diameter (0.02–0.024 m) on stress distribution in the rock masses.

Additionally, field experiments were conducted to validate the simulation results, with real-world measurements taken in operational mine environments. Comparative analyses were performed to assess the correspondence between model predictions and field data. Similar approaches were used by Ivanov [4] in a study on the behavior of rock masses under

dynamic loading conditions, which also integrated numerical modeling with real-world data to improve the reliability of geomechanical assessments in mining operations.

Furthermore, field and simulation-based validation studies by Zaitsev [5] contributed to the accuracy of stress-strain modeling. Zaitsev's research emphasized the importance of aligning theoretical models with empirical data for effective mining support system design. His findings underlined the critical role of simulation techniques in predicting the behavior of rock masses and optimizing mining support structures.

This methodology enables the detailed analysis of stress distribution and support system efficiency, ensuring safer mining operations and better resource management.

Results and Discussion

The research revealed the following insights into the behavior of mine workings: Longitudinal stresses increase with longer and thicker anchors, reflecting improved stability in resisting applied forces. However, excessive anchor dimensions may lead to diminishing returns in efficiency. This relationship has been previously discussed in the work of Kaiser and Tanant [6], who noted that while increasing anchor size improves stress resistance, there are limits to the effectiveness of larger anchors under certain conditions.

Shear stresses showed limited sensitivity to anchor length but slightly decreased with increasing anchor diameter, suggesting an indirect effect of anchor geometry on stress redistribution. This finding is supported by Zhang [7], who examined the role of anchor geometry in redistributing shear stresses in underground mine workings.

Vertical stresses exhibit moderate growth depending on the thickness of roof strata. Roof thickness directly influences load distribution and stress concentration in supporting structures. Zhang et al. [8] also observed that roof thickness significantly impacts the overall stress distribution in mine workings, highlighting its critical role in support design.

The analysis also revealed that metal arch supports are more resistant to shear stresses, while rock bolt supports are better at managing longitudinal stresses. This differentiation allows for tailored support strategies in different geological settings. This approach has been validated by Lee et al. [9], who demonstrated that different support systems perform better under specific types of stresses in various geological environments.

Field tests confirmed the validity of simulation models, with observed stress distributions closely matching predictions. The field validation of these models supports the findings of Zhang et al. [6], who successfully applied similar simulation techniques to predict stress behavior and optimize support systems in underground mines.

Theoretical Foundations of the Issue

The stability of mine workings is a multifaceted problem involving geological, mechanical, and technological factors. The stress-strain state of the rock masses surrounding mine workings is influenced by mining depth, rock properties, and support configurations. The theory of stress-strain states provides a foundation for understanding and predicting zones of plastic deformation and fracturing in rock masses. Applying this theoretical framework allows for the design of support

systems tailored to specific geological conditions. Advances in computational modeling have further enhanced the ability to simulate complex interactions within rock masses, leading to more effective solutions for mine stability challenges.

At the JSC «Qarmet» mines, where multiple production sections are in operation simultaneously, the accelerated and timely preparation of longwall faces is essential for increasing production volumes. Delays in preparation can lead to significant costs for repairs, both before and after commissioning [10].

Currently, the types of metal arch supports used are relatively expensive and lack technological efficiency, which negatively impacts development speed and maintenance conditions. This is primarily due to insufficient understanding of the behavior of the surrounding rock mass, as well as the imperfections in the design of the applied supports and installation technologies [11].

Maintaining and increasing the volume of underground coal mining is only feasible with the availability of highly efficient technologies for preparatory and maintenance operations, which ensure the growth of mine preparation volumes [12].

The objective of this research is to develop a technology for intensive and safe mining operations, based on identified patterns of behavior in the adjacent rock masses. This includes optimizing the parameters of technological schemes for preparatory operations to improve the efficiency of underground mining production. The core idea of the study lies in utilizing the induced stress-strain state of the rock mass to design an effective support technology for the contour-adjacent rock mass.

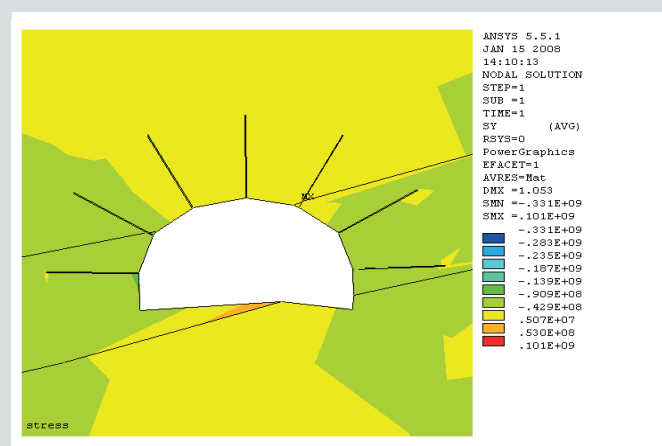


Figure 1. Manifestations of Rock Pressure in the Ventilation Drift Under a Reversible Ventilation Scheme for Longwall Mining (Cross-Section Along the Conveyor Drift).

Сурет 1. Тау жүктемесінің көріністері желдету үңгірінде, қайтармалы желдету схемасы бойынша қазба жұмыстарын жүргізу кезінде (конвейерлік қазбаның кесіндісі бойынша).

Рис. 1. Проявления горного давления в вентиляционной выработке при технологии очистных работ с возвратноточной схемой проветривания (разрез вкрест простирания по конвейерной выработке).

Horizontal displacements (U_x) on all sides are 0,35 m. Thus, the anchor support, even before the approach of the face, cannot withstand the acting pressure and requires reinforcement installation.

The conveyor excavation (see Figure 1), reinforced with anchor support, maintains stability before the face approaches, with tensile stresses of $b_y = 5$ MPa in the roof (soil) and compressive stresses $b_y = 50$ MPa in the sides of the excavation.

The conducted studies on determining the influence of roof rock control (the ratio of the thickness of the immediate roof rock to the extracted seam thickness) for metal-arch and anchor supports showed that, in general, as the roof rock thickness increases, all stresses exhibit a low-intensity growth dynamic.

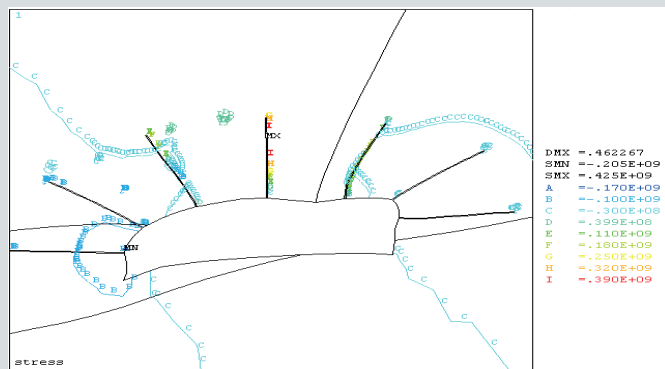


Figure 2. Manifestations of rock pressure in the ventilation drift (σ_y).

Сурет 2. Желдеткіш шығымы бойынша тау қысымының көрінісі (σ_y).

Рис. 2. Проявления горного давления на вентиляционной выработке (σ_y).

For any of the considered types of mine support, with the improvement of the controllability of the surrounding rocks, the stresses in the mass increase in a linear dependence. Vertical stresses (σ_y) increase insignificantly for both arch and anchor supports and are similar in magnitude.

Longitudinal stresses (σ_x) for the arch support are half as much compared to the anchor support, while shear stresses are four times greater. For metal-arched support, higher values are characteristic for shear stresses ($\tau_{xy} = 120\text{--}140$ Pa), while for the anchor support, they are more significant for longitudinal stresses ($\sigma_x = 60\text{--}70$ Pa) with approximately the same magnitude for their respective accompanying stresses in the range of 30–40 Pa, and minimal normal stresses (3–10 Pa) – as shown in Figure 2.

The stress-deformed state of the surrounding rock mass around the excavation was modeled with variations in the length and diameter of the anchor support. The effect of anchor length on the change in stresses in the rock mass was studied. The anchor length (ranging from 1.8 to 2.4 meters) had little effect on shear stresses, while vertical and longitudinal stresses increased slightly with the length of the anchor.

With a change in the diameter of the anchor (ranging from 0.02 to 0.024 meters), vertical and longitudinal stresses increased, while shear stresses decreased, following a nearly linear dependence. It was found that, in both cases, with an in-

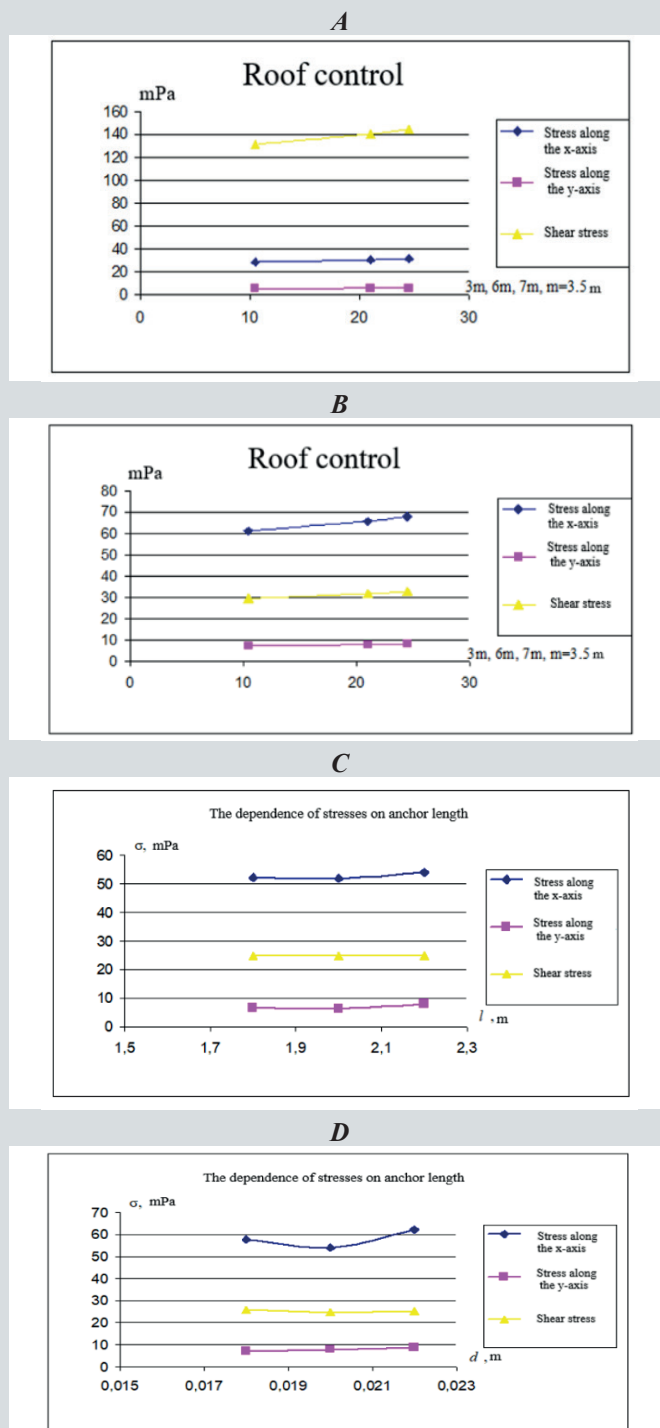


Figure 3. Influence of roof rock control on the stress magnitude around the contour of the excavation supported by metal-arch (A) and anchor (B) supports with changes in anchor length (C) and diameter (D) of the rod – anchor.

Сурет 3. Қабаттың жабынын басқарудың әсері, металл-қажетті (А) және анкерші (В) қаптамалармен бекітілген, ұзындығы (С) және таяқша диаметрі (D) өзгерген кезде пайда болатын кернеудің шамасы.

Рис. 3. Влияние управляемости пород кровли на величину напряжений, возникающих вокруг контура выработки, закрепленной металлоарочной (А) и анкерной (В) крепью с изменением ее длины (С) и диаметра (D) стержня – анкера.

crease in the length of the anchor (from 1.8 to 2.2 meters) and its diameter (from 0.02 to 0.024 meters), longitudinal stresses (55–60 Pa) were more significant, with a tendency for them to increase. Shear stresses remained almost unchanged (25 Pa) in the considered range, while normal stresses increased slightly in a linear dependence (from 5 to 10 Pa).

The conducted studies allowed determining the influence of technological factors on the effectiveness of applying metal-arch and anchor supports for mining workings. The identified patterns of change in the stress-deformed state of coal rock masses (displacements, stresses, and fracture zones) depending on the main mining-geological and mining-technical factors will allow for setting support parameters in specific operating conditions to improve the stability of preparatory mine workings. This will enable the development of new and the improvement of existing technologies for the effective and safe implementation of mine workings on gently sloping and inclined coal seams, adaptable to changing mining-geological and mining-technical conditions of operation.

A comparative evaluation of the conducted studies with tests in industrial conditions showed satisfactory convergence of the parameters of the stressed-deformed state of the rock mass.

Conclusion

The conducted research emphasizes the critical importance of understanding and managing the stress-strain state (SSS) of rock masses for the safe and efficient operation of underground coal mines. By integrating advanced numerical mod-

eling and field validation, this study identifies key patterns in the behavior of rock masses and their interaction with support systems. The findings reveal that both the length and diameter of anchor supports significantly influence longitudinal and vertical stresses, while having limited effects on shear stresses. Furthermore, metal-arch supports exhibit higher resistance to shear stresses, whereas anchor supports are more effective at handling longitudinal stresses.

The research highlights the necessity of tailoring support parameters to specific geological and technical conditions to optimize the stability of mine workings. This approach allows for the development of adaptive technologies that ensure efficient excavation processes, even in complex mining environments.

Ultimately, the study contributes to the ongoing advancement of underground mining operations by offering insights into the design and implementation of effective support systems. The results will aid in improving the safety and productivity of mining processes, supporting the growing demand for coal production while mitigating operational risks.

Acknowledgments

This research was funded under the program-targeted financing of the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan within the framework of the scientific and scientific-technical program IRN BR24992803 «Development of a rational mining technology based on the impact on the anthropogenic state of the enclosing rock mass».

REFERENCES

1. Protosenya A.G., Belyakov N.A., Buslova M.A. Modelirovanie napryazhenno-deformirovannogo sostoyaniya blochnykh massivov porod mestorozhdenii rud s primeneniem metodov obrusheniya [Modeling the Stress-Strain State of Block Rock Masses of Ore Deposits Using Caving Mining Methods], Zhurnal gornogo instituta [Journal of Mining Institute]. 2023. V. 262. 187–195 pp. (in Russian)
2. Gospodarikov A.P., Zatsepin M.A. Matematicheskoe modelirovanie napryazhenno-deformirovannogo sostoyaniya massivov porod pri otrabotke pologikh mestorozhdenii [Mathematical Modeling of the Stress-Strain State of Rock Masses during Shallow-Angle Deposit Development], Zhurnal gornogo instituta [Journal of Mining Institute]. 2010. V. 187. 88–92 pp. (in Russian)
3. Kreplenie gornykh vyrabotok na shakhtakh i kar'erakh [Mine Support for Underground Workings in Mines and Quarries]: A Textbook, edited by V.A. Kuznetsov: Perm: Permskii natsional'nyi issledovatel'skii politekhnicheskii universitet, 2016. 120 p. (in Russian)
4. Ivanov O.S. Primenenie geomekhaniki v gornykh rabotakh [Application of Geomechanics in Mining Operations], Novosibirsk: Sibirskoe otделение Rossiiskoi akademii nauk, 2019. 210–250 pp. (in Russian)
5. Zaitsev N.P. Tekhnologii krepleniya gornykh vyrabotok i geomekhanika [Technologies of Mining Fastening and Geomechanics], Yekaterinburg: Ural'skii universitet, 2017. 170–190 pp. (in Russian)
6. Kaiser P.K., Tannant D.D. Geomechanics for Underground Construction // Engineering Geomechanics: Toronto: 2017. 220–260 pp. (in English)
7. Zhang M. Geomechanics of Mining Operations: Beijing: Scientific and Technical Publishing House, 2017. 101–140 pp. (in Chinese)
8. Zhang L., Li H., Chen H. Rock Mechanics and Geotechnical Engineering: Amsterdam: Elsevier, 2019. 200–240 pp. (in English)
9. Lee H., Lee Y., Kim S. Evaluation of Support System Effectiveness in Underground Mining: Numerical and Field Testing Approach // Journal of Rock Mechanics and Geotechnical Engineering. 2018. V. 10. 377–386 pp. (in English)

10. Tikhomirov A.N. *Geomekhanika porod [Geomechanics of Rocks]*, Moscow: Geologiya i geomekhanika, 2015. 120–150 pp. (in Russian)
11. Petrov V.V., Morozov I.G. *Osnovy geomekhaniki [Fundamentals of Geomechanics]*, St. Petersburg: Gornaya Kniga, 2016. 200–230 pp. (in Russian)
12. Ivanov O.S. *Primenenie geomekhaniki v gornykh rabotakh [Application of Geomechanics in Mining Operations]*, Novosibirsk: Sibirskoe otделение Rossiiskoi akademii nauk, 2019. 210–250 pp. (in Russian)

ПАЙДАЛАНЫЛҒАН ӘДЕБИЕТТЕР ТІЗІМІ

1. Протосеня А.Г., Беляков Н.А., Буслова М.А. Рудалы кен орындарының блоктық тау жыныстары массивтерінің деформацияланған кернеу күйін құлау әдістерін қолдана отырып модельдеу // Тау-кен институтының журналы. 2023. Т. 262. Б. 187–195 (орыс тілінде)
2. Господариков А.П., Зацепин М.А. Жазық кен орындарын өңдеу кезінде тау жыныстары массивтерінің деформацияланған кернеу күйін математикалық модельдеу // Тау-кен институтының журналы. 2010. Т. 187. Б. 88–92 (орыс тілінде)
3. Шахталар мен карьерлердегі тау-кен қазбаларын бекіту: оқу құралы, редакциясын басқарған В.А. Кузнецов: Пермь: Пермь ұлттық зерттеу политехникалық университеті, 2016. 120 б. (орыс тілінде)
4. Иванов О.С. Тау-кен жұмыстарында геомеханиканы қолдану: Новосибирск: Ресей ғылым академиясының Сібір бөлімшесі, 2019. Б. 210–250 (орыс тілінде)
5. Зайцев Н.П. Тау-кен қазбаларын бекіту технологиялары және геомеханика: Екатеринбург: Урал университеті, 2017. Б. 170–190 (орыс тілінде)
6. Кайзер П.К., Таннант Д.Д. Жерасты құрылысы үшін геомеханика // Инженерлік геомеханика: Торонто: 2017. Б. 220–260 (орыс тілінде)
7. Чжан М. Тау-кен жұмыстарының геомеханикасы: Пекин: Ғылыми-техникалық баспа, 2017. Б. 101–140 (қытай тілінде)
8. Чжан Л., Ли Х., Чэнь Х. Тау жыныстары механикасы және геотехникалық инженерия: Амстердам: Эльзевир, 2019. Б. 200–240 (ағылшын тілінде)
9. Ли Х., Ли У., Ким С. Жерасты тау-кен жұмыстарында бекіту жүйелерінің тиімділігін бағалау: сандық және далалық сынақ әдісі // Тау жыныстары механикасы және геотехникалық инженерия журналы. 2018. Т. 10. Б. 377–386 (ағылшын тілінде)
10. Тихомиров А.Н. Тау жыныстарының геомеханикасы: М.: «Геология және геомеханика» баспасы, 2015. Б. 120–150 (орыс тілінде)
11. Петров В.В., Морозов И.Г. Геомеханика негіздері: СПб: Тау-кен кітабы, 2016. Б. 200–230 (орыс тілінде)
12. Иванов О.С. Тау-кен жұмыстарында геомеханиканы қолдану: Новосибирск: Ресей ғылым академиясының Сібір бөлімшесі, 2019. Б. 210–250 (орыс тілінде)

СПИСОК ИСПОЛЬЗОВАННЫХ ИСТОЧНИКОВ

1. Протосеня А.Г., Беляков Н.А., Буслова М.А. Моделирование напряженно-деформированного состояния блочных массивов пород месторождений руд с применением методов обрушения // Журнал горного института. 2023. Т. 262. С. 187–195 (на русском языке)
2. Господариков А.П., Зацепин М.А. Математическое моделирование напряженно-деформированного состояния массивов пород при отработке пологих месторождений // Журнал горного института. 2010. Т. 187. С. 88–92 (на русском языке)
3. Крепление горных выработок на шахтах и карьерах: учебник под ред. В.А. Кузнецова: Пермь: Пермский национальный исследовательский политехнический университет, 2016. 120 с. (на русском языке)
4. Иванов О.С. Применение геомеханики в горных работах: Новосибирск: Сибирское отделение Российской академии наук, 2019. С. 210–250 (на русском языке)
5. Зайцев Н.П. Технологии крепления горных выработок и геомеханика: Екатеринбург: Уральский университет, 2017. С. 170–190 (на русском языке)
6. Кайзер П.К., Таннант Д.Д. Геомеханика для подземного строительства // Инженерная геомеханика: Торонто: 2017. С. 220–260 (на английском языке)
7. Чжан М. Геомеханика горных работ: Пекин: Научно-техническое издательство, 2017. С. 101–140 (на китайском языке)
8. Чжан Л., Ли Х., Чэнь Х. Механика горных пород и геотехническая инженерия: Амстердам: Эльзевир, 2019. С. 200–240 (на английском языке)

9. Ли Х., Ли У., Ким С. Оценка эффективности систем крепления в подземных горных работах: подход на основе численных и полевых испытаний // Журнал механики горных пород и геотехнической инженерии. 2018. Т. 10. С. 377–386 (на английском языке)
10. Тихомиров А.Н. Геомеханика пород: М.: Геология и геомеханика, 2015. С. 120–150 (на русском языке)
11. Петров В.В., Морозов И.Г. Основы геомеханики: СПб: Горная книга, 2016. с. 200–230 (на русском языке)
12. Иванов О.С. Применение геомеханики в горных работах: Новосибирск: Сибирское отделение Российской академии наук, 2019. С. 210–250 (на русском языке)

Information about the authors:

Zhurlov V.V., Candidate of Engineering Sciences, Acting Associate Professor of the Higher Mathematics Department, Abylka Saginov Karaganda Technical University (Karaganda, Kazakhstan), zhurvitv@yandex.ru; <https://orcid.org/0000-0002-4413-8584>

Musin R.A., Ph.D., Acting Associate Professor, Abylka Saginov Karaganda Technical University (Karaganda, Kazakhstan), r.a.mussin@mail.ru; <https://orcid.org/0000-0002-1206-6889>

Zamaliyev N.M., Ph.D., Acting Associate Professor, Abylka Saginov Karaganda Technical University (Karaganda, Kazakhstan), nailzamaliyev@mail.ru; <https://orcid.org/0000-0003-0628-2654>

Akhmatnurov D.R., Ph.D., Head of Laboratory, Abylka Saginov Karaganda Technical University (Karaganda, Kazakhstan), d.akhmatnurov@gmail.com; <https://orcid.org/0000-0001-9485-3669>

Авторлар туралы мәліметтер:

Жұров В.В., т.ғ.к., Абылқас Сагинов атындағы Қарағанды техникалық университетінің жоғары математика кафедрасының міндетін атқарушы доценті (Қарағанды қ., Қазақстан)

Мусин Р.А., Ph.D докторы, Абылқас Сагинов атындағы Қарағанды техникалық университетінің міндетін атқарушы доценті (Қарағанды қ., Қазақстан)

Замалиев Н.М., Ph.D докторы, міндетін атқарушы доцент, Абылқас Сагинов атындағы Қарағанды техникалық университеті (Қарағанды қ., Қазақстан)

Ахматнуров Д.Р., Ph.D докторы, зертхана меңгерушісі, Абылқас Сагинов атындағы Қарағанды техникалық университеті (Қарағанды қ., Қазақстан)

Информация об авторах:

Жұров В.В., к.т.н., исполняющий обязанности доцента кафедры высшей математики Карагандинского технического университета им. Абылкаса Сагинова (г. Караганда, Казахстан)

Мусин Р.А., Ph.D, исполняющий обязанности доцента Карагандинского технического университета им. Абылкаса Сагинова (г. Караганда, Казахстан)

Замалиев Н.М., доктор Ph.D, исполняющий обязанности доцента, Карагандинский технический университет им. Абылкаса Сагинова (г. Караганда, Казахстан)

Ахматнуров Д.Р., доктор Ph.D, руководитель лаборатории, Карагандинский технический университет им. Абылкаса Сагинова (г. Караганда, Казахстан)